

# Implementation of an Actuated Inverted Pendulum Using a Real-time System

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**Abstract**—A micro robot that can be swallowed with sensors to inspect the intestine without the need to cut open a patient must have a propulsion mechanism to direct it to the areas of interest. The use of a propulsion mechanism external to the pill could endanger the patient, what is proposed here is a method of propelling a micro (capsule) robot with an internal mechanism based on the inertia of a swinging mass. The dynamics and control issues of a pendulum-driven cart-pole system [1] are similar to those of a capsule robot [4]. Therefore, this paper reports an experimental study of a pendulum-driven cart-pole system via a physical model.

**Index Terms**—micro-robot, propulsion, medical inspection

## I. INTRODUCTION

AN inverted pendulum driven cart system is investigated and built to assess its performance. The system is driven by the inertia of an accelerating inverted pendulum. The input to the system is a changing torque profile input to the pendulum driving motor. The pendulum angle and cart progress are monitored through the use of encoders connected to a PC. Two methods of control are investigated here: Method 1 is the application of a previous simulation using open loop control [3] which is shown to be unstable and sensitive to uncertainties with modelling, Method 2 is using a method of closed loop control which is shown to be stable and robust. The aim of this paper is to demonstrate the application of a real time system to controlling such a cart.

Initial investigations of an inverted pendulum driven cart demonstrated the feasibility of such a propulsion mechanism [1] and [2]. However, these systems are controlled by an open-loop control method and lack the power to alter the controlling torque profile and monitor the results. Simulations [3] have demonstrated that the open-loop control method is unstable. A fast, powerful and flexible real time controller is required. In the previous examples, an embedded controller is utilised, this gives the speed (high sample rate) but not the flexibility to accurately control the torque profiles, apply various control techniques, or gather data from multiple points on the cart.

The personal computer (PC) is a powerful and flexible computer but is often overlooked as a controller due to its slow sample rate and non-deterministic processes due to the common Microsoft Windows operating system (this software has many interrupts and handles a multitude of processes and thus a specific sample rate cannot be guaranteed). The PC

running Windows is also difficult to interface to input / output cards due to memory protection issues.

A solution is to use the National Instrument's real-time deployment operating system [5]. The PC bootstraps with this operating system, and the programs are written in Labview [6]. This graphical programming language simplifies the interface issues with the input/output cards.

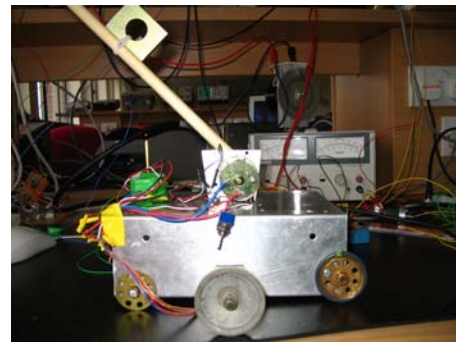


Figure 1 The cart system

## II. THE HARDWARE

### A. The Cart System

The cart is built with an aluminium base and low friction passive wheels for stability (figures 1 and 2). A central wheel which is not used for stabilization is connected to an encoder for positioning monitoring. A motor is fixed centrally on the chassis and is driven through a 30:1 planetary gearbox.

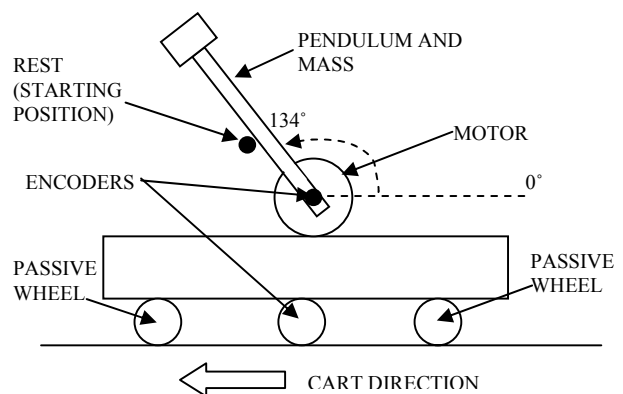


Figure 2 Schematic of the cart system

An encoder is connected directly to the gearbox shaft of the motor for angle monitoring. All electrical connections are routed via an umbilical cord with narrow gauge wire so as not to impede lateral motion.

The total mass of the cart with motors and encoders is 923g. The motor drives a pendulum having a length of 0.165m. A mass of 119g is connected to the end of the pendulum. The initial angle of the pendulum is 134 degrees.

Static and dynamic friction tests were conducted on the cart system placed on the rubber matting by using a spring balance. The cart was pulled by the spring balance until it started to move. Friction can be calculated using:

$$\mu = \frac{F_{\text{SPRING}}}{F_{\text{NORMAL}}}$$

The force required to move the cart was 0.14N, and the force required to keep it at a constant velocity was 0.06N. Knowing the normal force of the cart to be 10.22N (from its mass), the static and dynamic friction coefficients were found to be  $\mu=0.014$  and  $\mu=0.0059$  respectively.

The minimum torque required to lift the pendulum from 0° is calculated from:

$$T_m = F \times l$$

Where  $T_m$  is the torque delivered by the motor,  $F$  is the force exerted by the mass due to gravity, and  $l$  is the pendulum length. This minimum torque is therefore 0.193 Nm. Simulation from [3] has shown that sufficient progress can be made with a maximum torque of 0.35 Nm. The maximum velocity of the pendulum is 120° in 1500ms, or, 12.5ms / degree. The chosen sample rate is therefore 10ms which represents movement of less than 1°.

### B. Controller

The controller must be able to update a torque in the range of 0-0.35 N.m to the motor at a rate of 10ms and to control the motor to run in both directions. The chosen controller was a PC running the National Instrument's Labview Real Time Operating System 8.0. This allows the PC to process instructions and send data to an output card with a sample rate of up to 1us and is guaranteed to be deterministic (the samples are guaranteed to occur within a specified timeframe). Since the PC is running a real time operating system, there is no monitor or keyboard I/O, the PC communicates solely via an I/O card and the Ethernet port. A secondary PC is configured as a host terminal and is connected to the real time system through this Ethernet cable. This secondary computer acts as the visual interface through which the torque profiles can be modified and sent to the real time system and process the data from the real time system allowing the results to be observed and gathered on disk.

### C. Interface

There are a total of two 500ppr encoders on the system, one monitoring the pendulum angle and another monitoring the cart position. A motor driving a pendulum with an attached mass is attached to the cart. Interfacing with two encoders and

the driving motor is accomplished by using an I/O card [7], a digital acquisition (DAQ) National Instruments 6221 PCI M-series 16 bit, 250 KS/s card having two analogue output channels and two encoder counter channels. The card is interfaced to the real time system, the encoders can be read directly via the counters and the motor is connected to the analogue output channel via a voltage to current amplifier.

### D. Motor driver

What is required is an accurate method of controlling the motor torque. This cannot be achieved by controlling the voltage to the motor due to the back EMF or voltage generated by the motor which is proportional to its velocity. Motor torque is proportional to the armature current ( $I_a$ ) regardless of the shaft velocity:

$$I_a = \frac{T_m}{K_t} \dots\dots\dots(1)$$

where  $T_m$  is the torque developed by the motor and  $K_t$  is the motor torque constant.

A constant current driver circuit was developed and improved from the Howland Current Source [8], the input is a voltage in the range of 0-3.4V and the output is a current in the range of 0-1.26A. This driver is able to drive a motor bi-directionally. A negative voltage drives the current to a negative value and turns the motor the opposite direction.

The operation of the circuit is as follows (figure 3): Operational amplifier U2 acts as a voltage buffer for voltage  $V_2$ . Operational amplifier U1 adds the voltages  $V_{IN}$  and  $V_2$  and outputs this to  $V_1$ :

$$V_1 = V_{IN} + V_2 \dots\dots\dots(2)$$

The resistor  $R_{REF}$  determines the current, since:

$$R_{REF} = \frac{V_1 - V_2}{I_a} = \frac{V_{IN}}{I_a} \dots\dots\dots(3)$$

and combining (3) with (1) gives the relationship:

$$V_{IN} = \frac{T_m R_{REF}}{K_t} \dots\dots\dots(4)$$

The relationship of voltage to current is linear.

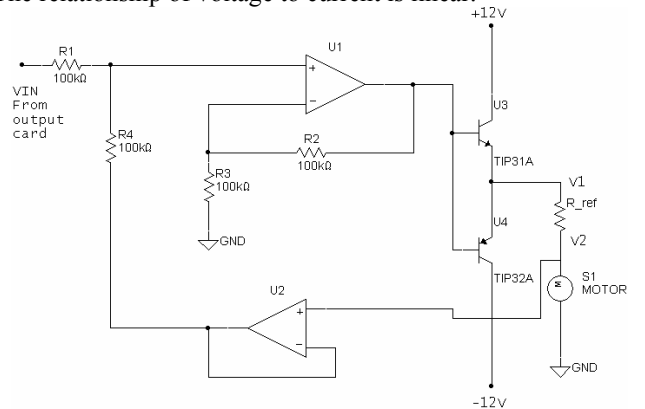


Figure 3 The constant current driver circuit

The motor's stationary torque was measured for the input voltages in the range of 0-3.24V by having the motor move a bar of length 0.3M resting on a digital scale having an accuracy of 1gram. The results were a linear relationship between input voltage and the developed torque as shown in figure 4 with the exception of the initial torque. This is due to the friction of the gearbox and was found to be 0.063N.m from the graph.

The relationship between the input voltage and the torque is found to be:

$$\frac{R_{REF}}{K_T} = 9.26, \text{ or:}$$

$$V_{IN} = 9.26 \times T_m$$

Since  $R_{REF}$  is fixed in the circuit at a value of  $0.76\Omega$ ,  $K_T$  has a value of  $0.081$ . From (1), the maximum torque we can expect from a 5A power supply is therefore  $0.405 \text{ N.m}$ , this is within the specified range.

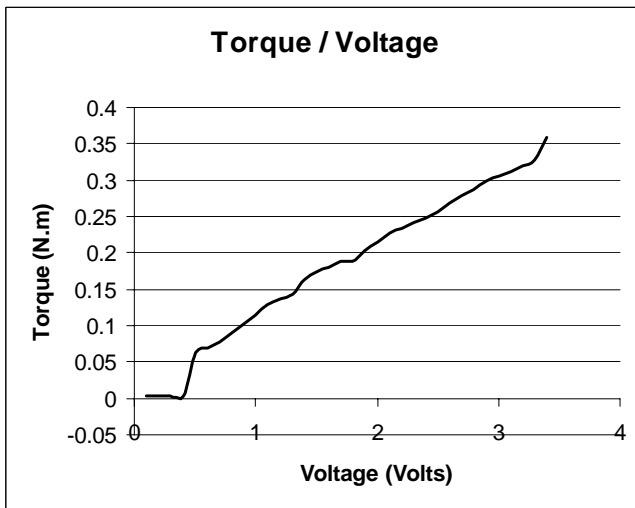


Figure 4 Voltage / torque ratio

The motor driver was connected to the output of the analogue output channel of the DAQ card.

The output from each of the encoders was connected to the A and B channels of counters 0 and 1 which is internal to the DAQ card.

#### E. Host computer

The host computer acts as a terminal for the real time system. It is through this host computer that the real time system is controlled, programmed, and results displayed (figure 5). The host is connected via an Ethernet cable to the real time system. The host computer sends the required torque values to the real time system in a single data burst. The real time system stores the data in an array and waits until all of the data is received before it begins its output.

The host can generate torque data in two ways: reading a text file of previously saved torque data, or by drawing the required torque output on a chart.

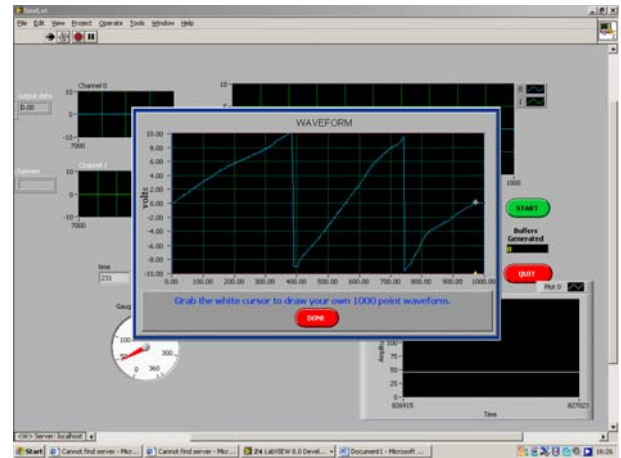


Figure 5 The host control software

The program running on the host PC loads the torque data from the simulation scales it to represent the desired voltage values for the driver circuit, and sends to the real time system. The real time system outputs this voltage to the motor driving the pendulum. The output from the encoders is sent back to the host for displaying on a waveform graph.

Two methods were employed for controlling the torque in the experiment:

Method 1 updated the torque profile against time, and the torque profile had been previously verified to give the necessary pendulum angles and acceleration in the simulation. Method 2 used the actual pendulum angle as an index to updating the next torque values with a distinction made between the forward traversing angles and the returning angles. In both cases, the torque profiles were initially loaded onto the real time system from the host, the difference between the two indexing methods was realised by altering the program on the real time system.

### III. RESULTS

#### A. Method 1- Open-loop control approach

The open-loop control approach is used for method 1 [3]. The control torque was input as in figure 6. This was calculated to force the pendulum down rapidly (point A), followed by a reverse torque to stop the pendulum (point B). This was followed by a decreasing reverse torque (point C) to ensure the pendulum returned to the start position without hitting any end-stops.

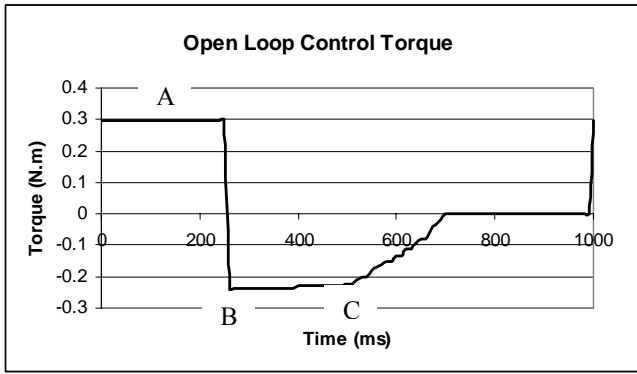


Figure 6 Open loop control torque

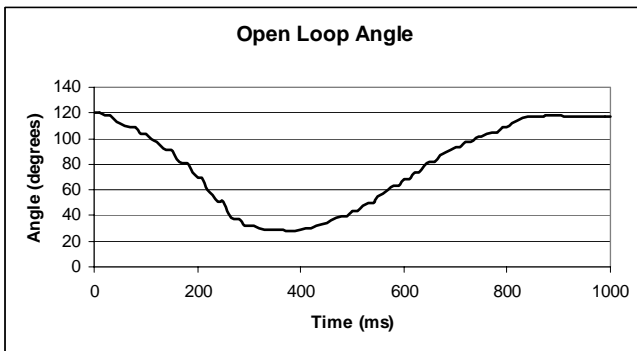


Figure 7 Open loop pendulum angle

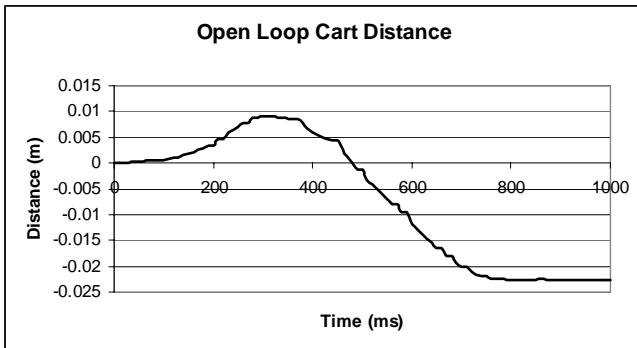


Figure 8 Open loop cart distance

The pendulum oscillated about its ‘action angles’ (figure 7) although the cart made little progress (figure 8), reaching a total of 23mm after one second. This was due to the uncontrollable nature of the pendulum, the pendulum did not necessarily return to its optimum position before recoiling as any disturbances to the cart motion would upset the pendulum position. Difficulty arose in gently bringing the pendulum to a halt and the side-effect was the cart slipped backwards in the opposite direction.

#### B. Method 2- closed loop control approach

Method 2 was not based on the assumption that the pendulum would be at a certain angle at a calculated time but rather used

the angle of the pendulum itself as a feedback mechanism to close the loop and index the next torque value. The closed-loop control approach used for method 2 is derived from the control experience in method 1 (figure 8). This happened in a decision loop following the angle format as shown in figure 11. By analysing the figure 8, the control area is divided into several regions. In each region, a constant torque is applied on the motor. Positive torque acts to move the pendulum fast (clockwise) in order to drive the cart. Negative torque acts to stop the cart and move the pendulum towards its initial starting position, i.e. anticlockwise.

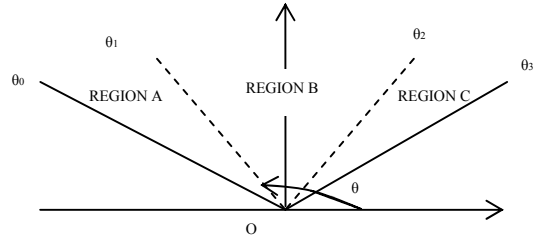


Figure 9 Regions used for torque control

#### 1) Experiment results

The new torque data (6) is applied in a computer program to output the torques for the angles.

$$\tau = \begin{cases} 0.35 & \theta(k) \in [180^\circ, 110^\circ) \\ 0.2 & \theta(k) \in [110^\circ, 40^\circ), \theta(k+1) - \theta(k) < 0 \\ -0.2 & \theta(k) \in [40^\circ, 0^\circ), \theta(k+1) - \theta(k) < 0 \\ -0.2 & \theta(k) \in [0^\circ, 90^\circ), \theta(k+1) - \theta(k) \geq 0 \\ -0.1 & \theta(k) \in [90^\circ, 120^\circ), \theta(k+1) - \theta(k) \geq 0 \end{cases} \quad (6)$$

This resulted in a smooth operation of the pendulum oscillating between 125° and 18° (figure 10).

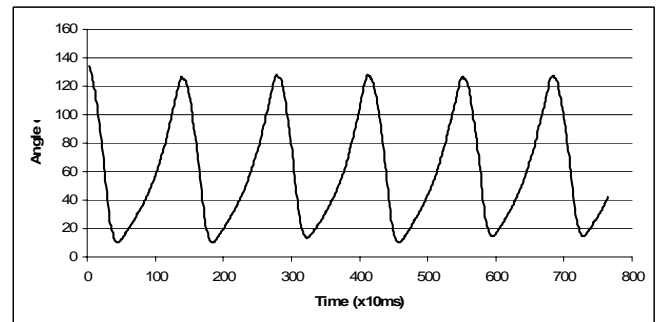


Figure 10 The pendulum angle from an angle indexed torque profile

The cart made a steady progress and reached 16 cm in 7.5 seconds (figure 11). The average speed is about 2.13 cm/s.

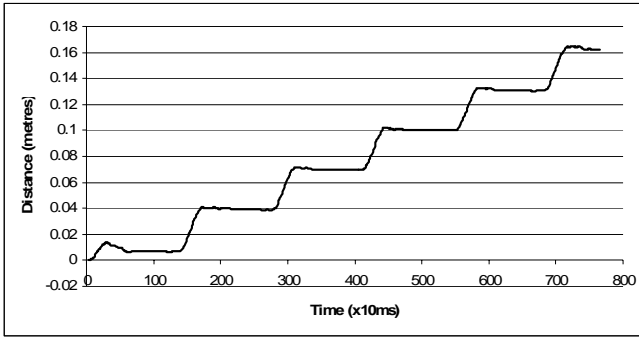


Figure 11 The cart position from a time indexed torque profile

### C. Detail of a single stroke cycle

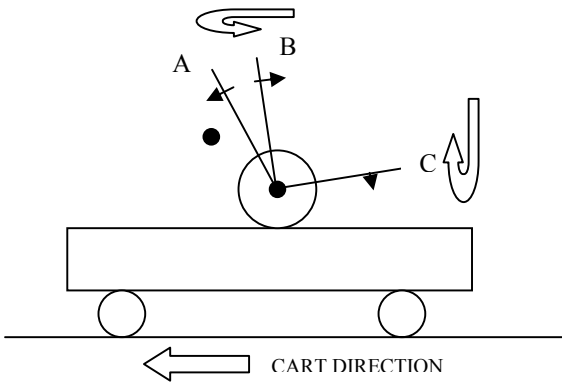


Figure 12 Pendulum directions and positions

A single stroke cycle was investigated (figure 12) and the speed is obtained by taking the average of the difference of the current and previous two cart positions.

From the graph of figure 13, point A shows the pendulum on its return stroke nearing its starting position, the change of direction is happening when the pendulum is almost vertical resulting in a force to the normal of this in the horizontal direction. Point C shows the pendulum reversing its stroke when almost horizontal resulting in a normal force on the vertical plane, the constraints of the surface prohibits its movement.

Figure 14 shows the results that the cart has been stationary for the return stroke and is coming to the end of its stationary period.

The pendulum reverses its direction as shown in point B figure 13. It is at this point of the pendulum reversing and accelerating in the opposite direction that the greatest velocity occurs (figure 15) and the cart covers the greatest distance (figure 14). The cart progresses until the pendulum is at its end of travel (point C) and the cart remains stationary until the next reversing pendulum stroke.

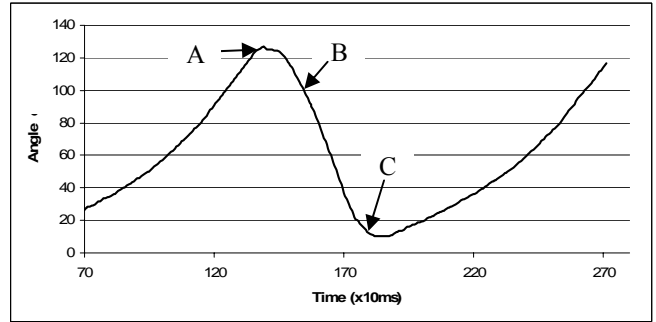


Figure 13 Single pendulum stroke

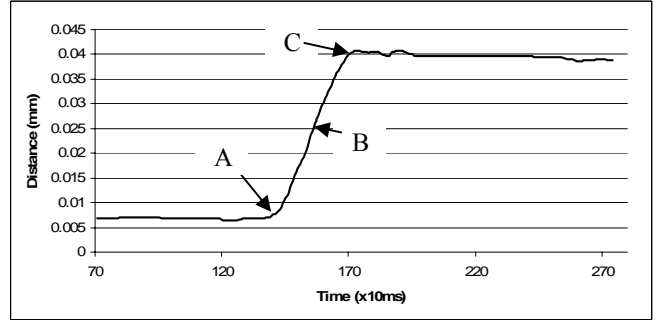


Figure 14 Cart movement during this single stroke

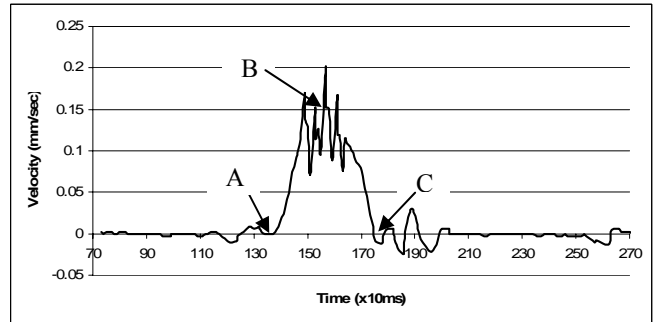


Figure 15 Cart velocity during single stroke

A different torque profile (7) was experimented by changing the torque values and the angle boundaries that each one is activated as follows

$$\tau = \begin{cases} 0.35 & \theta(k) \in [180^\circ, 110^\circ) \\ 0.2 & \theta(k) \in [110^\circ, 40^\circ), \theta(k+1) - \theta(k) < 0 \\ -0.3 & \theta(k) \in [40^\circ, 0^\circ), \theta(k+1) - \theta(k) < 0 \\ -0.3 & \theta(k) \in [0^\circ, 70^\circ), \theta(k+1) - \theta(k) \geq 0 \\ -0.2 & \theta(k) \in [70^\circ, 90^\circ), \theta(k+1) - \theta(k) \geq 0 \\ -0.1 & \theta(k) \in [90^\circ, 120^\circ), \theta(k+1) - \theta(k) \geq 0 \end{cases} \quad (7)$$

The results showed the cart made greater progress (taking 6 seconds to reach the previous distance of 18 cm) but also slipped back to a large degree on the return stroke (figure 16) due to the increased acceleration of the return stroke. This shows that the motion of the cart is controllable by altering the torque profile using angle as an index. It also shows that optimisation of the torque profile could result in a more efficient progress.

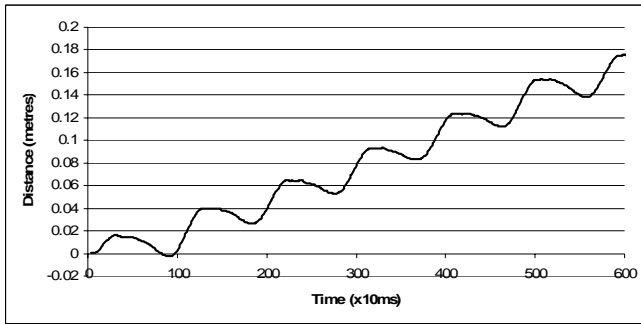


Figure 16 Updated cart progress

#### IV. CONCLUSIONS

A real time system was developed to provide robust and accurate control, and a motion rig constructed to demonstrate the motion of a cart controlled from an inverted pendulum. Solid tests were conducted and the results captured and analysed. The idea to drive the cart by swinging the inverted pendulum was proved by the real experiment.

Method 1 of using time as an index worked during the simulation. This however produced an unsatisfactory performance of pendulum swing in the working model due to unforeseen parameters in the real system that was not modelled. A more accurate measurement of static and dynamic friction is necessary as the frictions do not remain the same for different parts of the same surface.

The unforeseen parameters were minimised by the use of angle as a feedback mechanism in method 2. The closed loop system was more robust to these deviations. As time was not used as an index, the pendulum was not assumed to be in a certain place at a particular time. The pendulum reached a steady oscillation and the cart made most of its acceleration at the point of the pendulum reaching its return position from moving anti-clockwise and accelerating in a clockwise direction. The cart made a constant progress in the desired direction due to the tuning of the torque values and angles.

An accurate measurement of friction allowed the torque profile to be tailored to give the maximum progress. Most progress occurs when the pendulum reverses direction when it is in the upright position and the normal faces horizontally. The return swing causes a force in the normal plane to act vertically but, due to the constraints of the surface and friction, the cart does not slip back.

#### V. DISCUSSION

The use of a real time system in this work has demonstrated the flexibility and its usefulness in research work where parameters have to be accurately changed and results carefully monitored.

Further work will involve the optimisation of the torque profile depending on the desired movement. This optimisation would take place by having the program automatically alter the torque profile based on the feedback of the cart progress. Certain 'costs' could be included, i.e. minimum slip-back, maximum distance, faster movement, wheels not leaving the surface etc and will be the subject of further research. This demonstrates that adaptive control can be utilised in this propulsion method to give the desired progress along with the power of a real time system.

#### REFERENCES

- [1] Hongyi Li; Furuta, K.; Chernousko, F.L. "A Pendulum Driven Cart via Internal Force and Static Friction", Proceeding of the 2005 International Conference on Physics and Control, 24-26 Aug. 2005 Page(s):15 - 17
- [2] Wane S, Yu H, Yang T C, "Development of a reaction drive for a propulsion mechanism" IEEE International Conference on Networking, Sensing and Control, April 15-17 2007.
- [3] Y. Liu, H. Yu, and B. Burrows, "Optimization and Control of a Pendulum-driven Cart-pole System", Proc. of the 2007 IEEE International Conference on Networking, Sensing and Control, London, UK, April 2007.
- [4] Hongyi Li, Katsuhisa Furuta and F. L. Chernousko, "Motion Generation of the Capsbot Using Internal Force and Static Friction", Proc. of the 45<sup>th</sup> IEEE Conference on Decision & Control, San Diego, USA, December 2006.
- [5] National Instruments Real-time measurement, Jul 2007. [Online]. Available: <http://www.ni.com/realtime/>
- [6] National Instruments Labview programming language, Jul 2007. [Online]. Available: <http://www.ni.com/labview/>
- [7] National Instruments Application note, Jul 2007. [Online]. Available: <http://sine.ni.com/nips/cds/view/p/lang/en/nid/14132>
- [8] M. Hosseini, "Voltage controlled current source uses two op-amps", ElectronicDesign, [Online]. Available: <http://www.elecdesign.com/Articles/Index.cfm?AD=1&ArticleID=9018>